To predict the operating characteristics of a transformer design, one must be able to calculate the leakage inductance, distributed capacitance, and open circuit inductance. Following are outlined procedures for calculating these parameters.

6.3.1 Leakage Inductance

Equations for calculating the leakage inductance of a simple concentric winding will be developed first. Then corrections will be made for other winding configurations.

6.3.1.1

Figure 6.3-1 shows the cross section of a simple concentric transformer with the MMF diagram of the distributed ampere-turns due to the load current of the transformer.

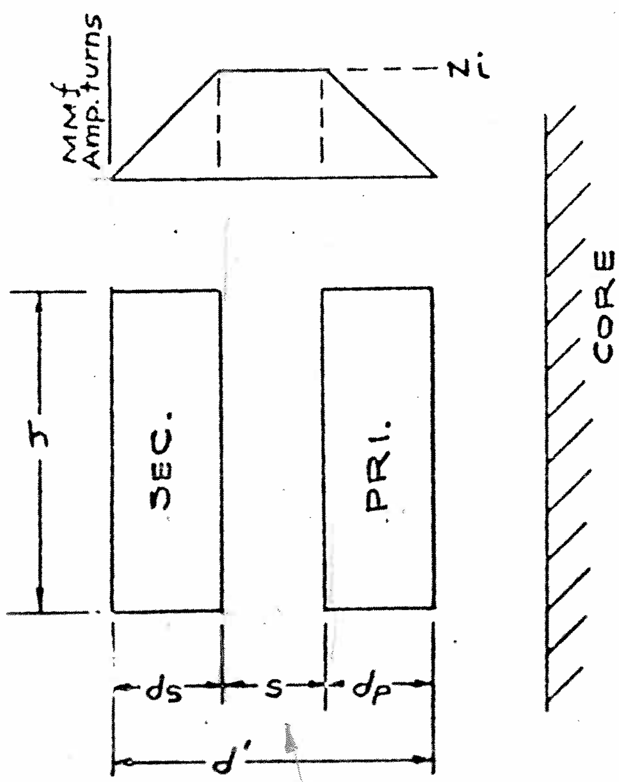


FIGURE 6.3-1

*The derivations of this section closely follow the derivation of percent reactance in reference 8.1.1.1.

The mmf producing the field at the pad (S) is the ampere turns of the primary, or the secondary winding. If we assume that the flux diverges rapidly after it leaves the coil, so that its density is reduced to a low value, the reluctance of the external path will be negligible, and the length of the leakage flux path will be the same as the winding traverse (h). Then the flux density in the pad will be:

$2.54(.4\pi)$

$H = \frac{.4\pi NI}{l_c} = B$

$$B = \frac{3.19\ Ni}{h} \quad \text{lines/sq.in.}$$

$B = \frac{.4\pi NI}{l_c}$ (cm) (IN/2.54)

Assuming a uniform flux density in the pad, the total flux in the pad is:

$$\Phi = BA_s = \frac{3.19\ Ni\,\ell_s S}{h} \quad \text{lines}$$

from Faraday's Law

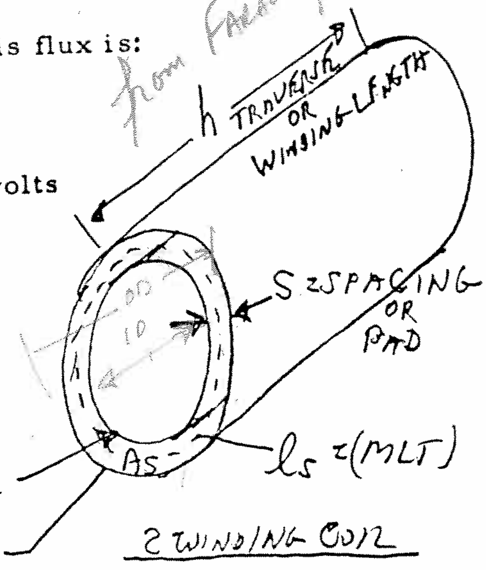
The peak voltage induced in a winding by this flux is:

$$e = 2\pi f N \Phi (10)^{-8}$$

BA

$$e = \frac{(2)(3.19)\pi f N^2 i\,\ell_s S(10)^{-8}}{h} \quad \text{volts}$$

$B = \frac{V\,10^8}{4.44\,ANf}$



The inductive reactance is:

$$X_L = \frac{e}{i} = 2\pi f L_L \quad \text{Ohms}$$

Solving for L_L, we get:

$$L_L = \frac{e}{2\pi f i} \quad \text{henries}$$

$\pi\sqrt{2}$

Substituting for e

$$L_L = \frac{3.19\ N^2 \ell_s S(10)^{-8}}{h} = \frac{3.19 N^2 A_s (10)^{-8}}{h} \qquad (6.3\text{-}1)$$

Where:

$$A_s = \ell_s S \quad \text{sq. inches}$$

Note that A_s is the cross sectional area of the pad S, and L_L is the leakage inductance between the primary and secondary windings, referred to the winding having (N) turns.

Equation (1) can be used to calculate the leakage inductance of a two winding, single coil transformer, where the radial build of the coils is negligible compared to the radial build of the insulation pad (S) between windings.

$$\frac{31.9\ \cancel{3.19}\ N^2 (MLT)(\text{build})}{W}\ nH$$

6.3.1.2

An expression for multiple pad coils can be obtained by finding the energy stored in each pad, and adding these to obtain the total stored energy.

The energy stored in the pad is:

$$\mathcal{E} = \frac{Li^2}{2} \text{ joules}$$

substituting for L from equation 6.3-1

$$\mathcal{E} = \frac{3.19\,(Ni)^2 \; A_s\,(10)^{-8}}{2\,h} \text{ joules} \qquad (6.3\text{-}2)$$

Equation 6.3-2 gives the energy stored in the electromagnetic field between any two coils where (N_i) is the peak ampere turns acting on the pad, (A_s) is the cross sectional area of the pad, and h is the winding traverse.

For multiple pad coils, the total energy stored is:

$$\mathcal{E} = \frac{3.19\,(N_1\,i_1)^2 A_1 (10)^{-8}}{2h} + \frac{3.19 (N_2\,i_2)^2 A_2 (10)^{-8}}{2h} + \;.\;.\;.\; + \frac{3.19\,(N_n i_n)^2 \; A_n (10)^{-8}}{2h} \text{ joules}$$

$$\mathcal{E} = \frac{3.19 (10)^{-8}}{2h} \left[(N_1\,i_1)^2 \; A_1 + (N_2\,i_2)^2 \; A_2 + \;.\;.\;.\; + (N_n i_n)^2 A_n\right]$$

$$\mathcal{E} = \frac{L_L\,i_s^2}{2} \text{ joules}$$

Where L_L is the leakage inductance referred to the winding carrying i_s peak amps.

Solving for the leakage inductance:

$$L_L = \frac{3.19 (10)^{-8}}{h\,i_s^2} \left[(N_1\,i_1)^2 \; A_1 + (N_2 i_2)^2 \; A_2 + \;.\;.\;.\; + (N_n\,i_n)^2 A_n\right] \text{henries}$$

Let N_s be the number of turns on the winding carrying i_s peak amps. Then, multiplying by (N_s^2/N_s^2):

$$L_L = \frac{3.19\,N_s^2 (10)^{-8}}{h} \left[\frac{(N_1\,i_1)^2 A_1}{(N_s\,i_s)^2} + \frac{(N_2\,i_2)^2}{(N_s\,i_s)^2} \; A_2 + \;.\;.\;.\; + \frac{(N_n i_n)^2 \; A_n}{(N_s i_s)^2}\right]$$

For any given wave shape, the ratio of two peak currents equals the ratio of the rms value of the same currents, or:

$$\frac{i_1}{i_s} = \frac{I_1}{I_s}$$

JASOND

Therefore:

$$L_L = \frac{3.19\, N_s^2 (10)^{-8}}{h} \left[\left(\frac{N_1 I_1}{N_s I_s}\right)^2 A_1 + \left(\frac{N_2 I_2}{N_s I_s}\right)^2 A_2 + \ldots + \left(\frac{N_n I_n}{N_s I_s}\right)^2 A_n \right]$$

$$L_L = \frac{3.19\, N_s^2 (10)^{-8} A_s}{h} \text{ henries} \qquad (6.3\text{-}3)$$

Where:

$$A_s = \left(\frac{N_1 I_1}{N_s I_s}\right)^2 A_1 + \left(\frac{N_2 I_2}{N_s I_s}\right)^2 A_2 + \ldots + \left(\frac{N_n I_n}{N_s I_s}\right)^2 A_n \qquad (6.3\text{-}4)$$

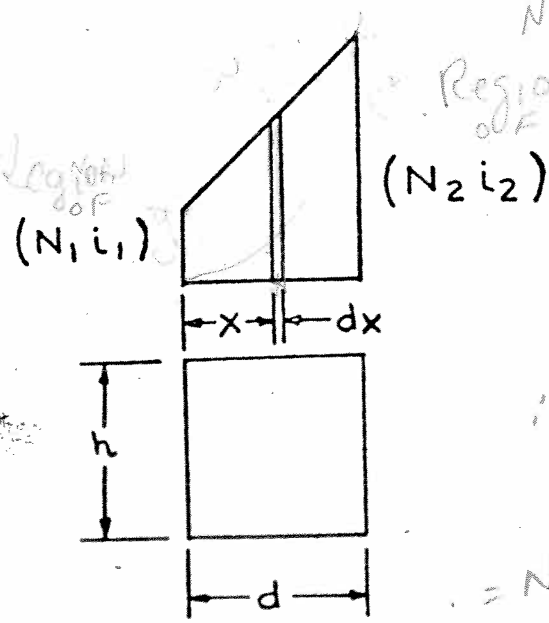
note $A_1 = MLT_1\, S_1$, $A_2 = MLT_2\, S_2$ etc.

Equation (6.3-3) gives the leakage inductance between primary and secondary referred to the winding with N_s turns. Note that $\frac{(N_n I_n)}{(N_s I_s)}$ is the fraction of the total ampere turns acting on the N^{th} gap, and A_n is the cross sectional area of that gap.

<u>6.3.1.3</u>

The development of equation 6.3-3 neglected the mmf distributed over the thickness of the coil. If the copper build, ds and dp in figure 6.3.1 is small when compared to the pad thickness S, equation 6.3-3 will give reasonably accurate results. When this is not true, the effect of the coil build must be considered.

Figure 6.3-2 shows the cross section of a coil and its associated mmf diagram. To make this a general case, it is shown as a coil in the middle of other coils. Thus, the mmf is not zero on either side of the coil.



Ni_x Region of $y = mx + b$

$N_1 i_1$

$= \frac{N_2 i_2 - N_1 i_1}{d}$

$\therefore \frac{N_2 i_2 - N_1 i_1}{d} x + N_1 i_1$

$= N_1 i_1 + N_2 i_2 \frac{x}{d} - N_1 i_1 \frac{x}{d}$

FIGURE 6.3-2

The ampere turns at (X) is:

$$(NI)_x = (N_1 i_1) + (N_2 i_2) \frac{X}{d} - (N_1 i_1) \frac{X}{d}$$

From equation (6.3-2), the energy stored at (X) is:

$$\mathcal{E}_x = \frac{3.19 \, (NI)_x^2 \, A_x \, (10)^{-8}}{2h} \text{ joules}$$

Substituting for $(NI)_x$ from above:

$$\mathcal{E}_x = \frac{3.19 \, \ell_c \, (10)^{-8}}{2\,h} \left[(N_1 i_1) + (N_2 i_2) \frac{X}{d} - (N_1 i_1) \frac{X}{d} \right]^2 d_x \text{ joules}$$

The energy stored in the coil build is:

$$\mathcal{E} = \frac{3.19 (10)^{-8} \ell_c}{2h} \int_0^d \left[(N_1 i_1) + (N_2 i_2) \frac{X}{d} - (N_1 i_1) \frac{X}{d} \right]^2 d_x$$

Integrating over the limits shown gives:

$$= \frac{3.19 \, (10)^{-8} \ell_c}{2h} \left[(N_1 i_1)^2 \frac{d}{3} + (N_1 i_1)(N_2 i_2) \frac{d}{3} + (N_2 i_2)^2 \frac{d}{3} \right] \text{ joules}$$

$$= \frac{L_L \, i_s^{\,2}}{2}$$

The leakage inductance due to the coil radial build is:

$$L_L = \frac{3.19 (10)^{-8} \ell_c}{h i_s^{\,2}} \left[(N_1 i_1)^2 + (N_1 i_1)(N_2 i_2) + (N_2 i_2)^2 \right] \frac{d}{3} \quad \text{henries}$$

Multiplying by $(N_s/N_s)^2$ gives:

$$L_L = \frac{3.19 (10)^{-8} N_s^2 \, \ell_c d}{h} \left[\frac{(N_1 i_1)^2 + (N_1 i_1)(N_2 i_2) + (N_2 i_2)^2}{3 (N_s i_s)^2} \right] \text{henries}$$

Since:

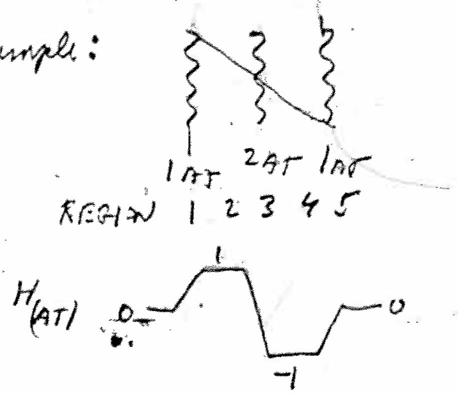
$$\frac{i_1}{i_s} = \frac{I_1}{I_s}$$

$$L_L = \frac{3.19 (10)^{-8} \, N_s^{\,2} \ell_c d}{h} \left[\frac{(N_1 I_1)^2 + (N_1 I_1)(N_2 I_2) + (N_2 I_2)^2}{3 \, (N_s I_s)^2} \right]$$

$$L_L = \frac{3.19 \, N_s^{\,2} \, A_w (10)^{-8}}{h} \quad \text{henries} \qquad \textbf{(6.3-5)}$$

Where $A_w = \ell_c d \left[\frac{(N_1 I_1)^2 + (N_1 I_1)(N_2 I_2) + (N_2 I_2)^2}{3 \, (N_s I_s)^2} \right]$

example:



FOR REGION 3

$$A_w = \ell_c d \left[\frac{1^2 + (-1 * 1) + (-1^2)}{3 (2^2)} \right]$$

$$A_w = \ell_c d \left[\frac{1 - 1 + 1}{3 (4)} \right] = \frac{1}{3} \left(\frac{1}{4} \right)$$

For transformers with multiple coils

$$A_w = \ell_j d_j \left[\frac{(N_jI_j)^2 + (N_jI_j)(N_{j+1}I_{j+1}) + (N_{j+1}I_{j+1})^2}{3(N_sI_s)^2} \right] \qquad (6.3\text{-}6)$$

Adding equations (6.3-3) and (6.3-5), the transformer leakage inductance is:

$$L_L = \frac{3.19\, N_s^2}{h} (A_s + A_w)(10)^{-8} \quad \text{henries} \qquad (6.3\text{-}7)$$

Where A_s and A_w are defined by equations (6.3-4) and (6.3-6) respectively, and L_L is the leakage inductance referred to the winding having (N_s) turns.

The accuracy of the assumption that the leakage flux path length is equal to the coil height depends on the coil height to coil build ratio (h and d' in figure 6.3-1). For multiple gap coils.

$$d' = \lambda/2$$

Where λ is the wave length of the mmf diagram, as shown in figure 6.3-3.

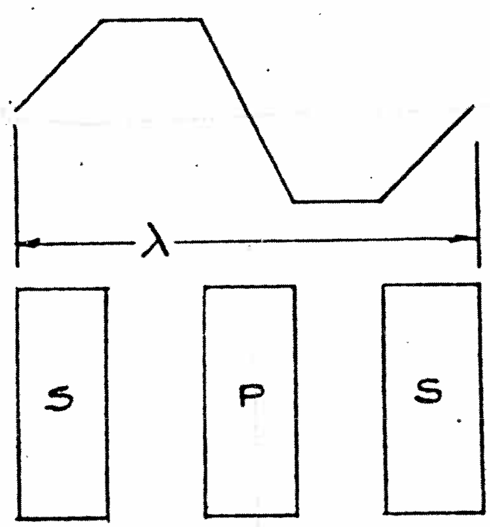


FIGURE 6.3-3

Considering the (h/d') ratio, the leakage inductance becomes:

$$L_L = \frac{K_r 3.19\, N_s^2}{h} (A_s + A_w)(10)^{-8} \text{ henries} \qquad (6.3\text{-}8)$$

Where K_r the Rogowski correction factor (reference 8.1.2.2) is:

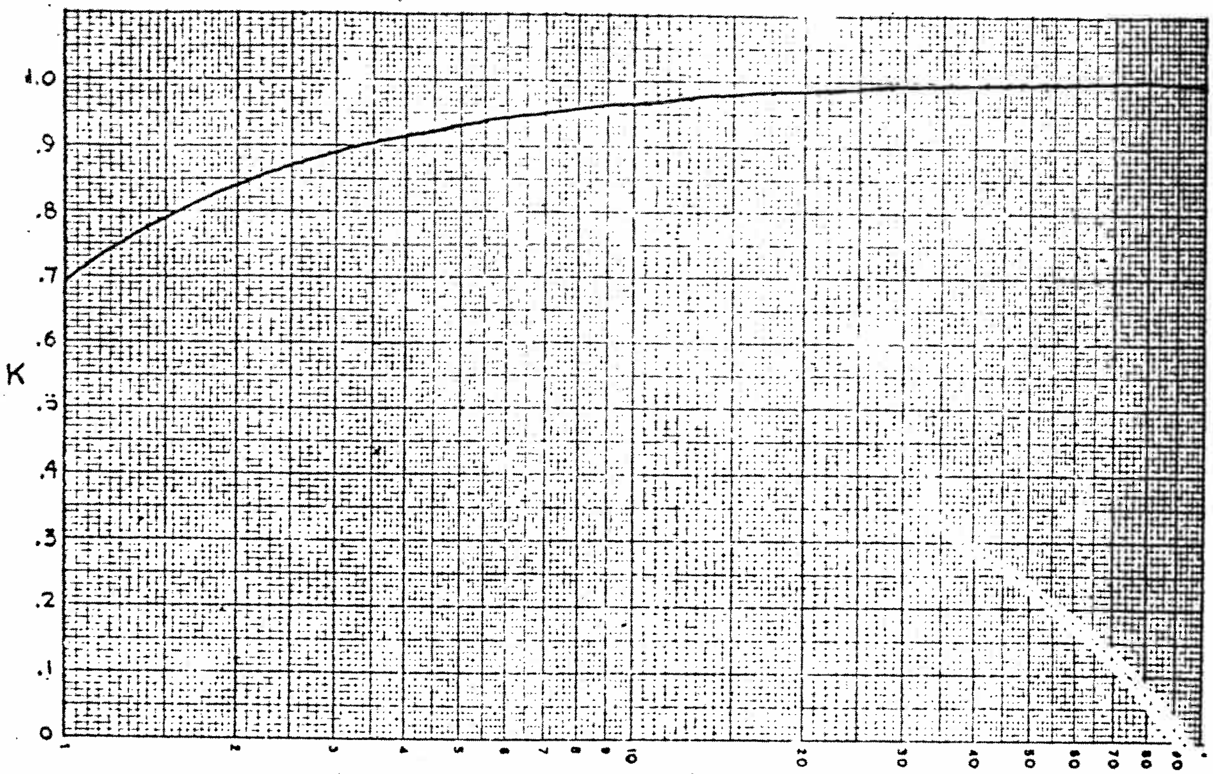
$$K_r = 1 - \frac{1 - \frac{\pi h}{\epsilon\, d'}}{\frac{\pi h}{d'}}$$

$\frac{1}{\frac{\pi h}{d'}} - \frac{1}{\epsilon}$

s/b $\frac{1}{K_r}$?

$$K_r = 1 + \left[\frac{2\pi\left(\frac{l_w}{Build}\right) - 1}{2\pi \frac{l_w}{Build}} \right] \quad ??$$

Figure 6.3-4 is a plot of K_r vs. the h/d' ratio.



$h/d' = \dfrac{l_{coil}}{Build_{total}}$

Rogowski Correction Factor

FIGURE 6.3-4

6.3.1.5

The derivation of equation (6.3-7) and (6.3-8) assume that all coils had the same traverse. If coils are of unequal lengths, or if there are tapped out sections, the leakage inductance calculation is more complicated.

A method of calculating the leakage inductance of transformers with windings of different lengths, or with tapped out sections has been devised by H.O. Stephens (reference 8.1.2.3). The following example will outline his method.

The leakage inductance of the transformer shown in figure 6.3-5 is to be calculated. The numbers in each coil is the per cent of total ampere turns in that coil.

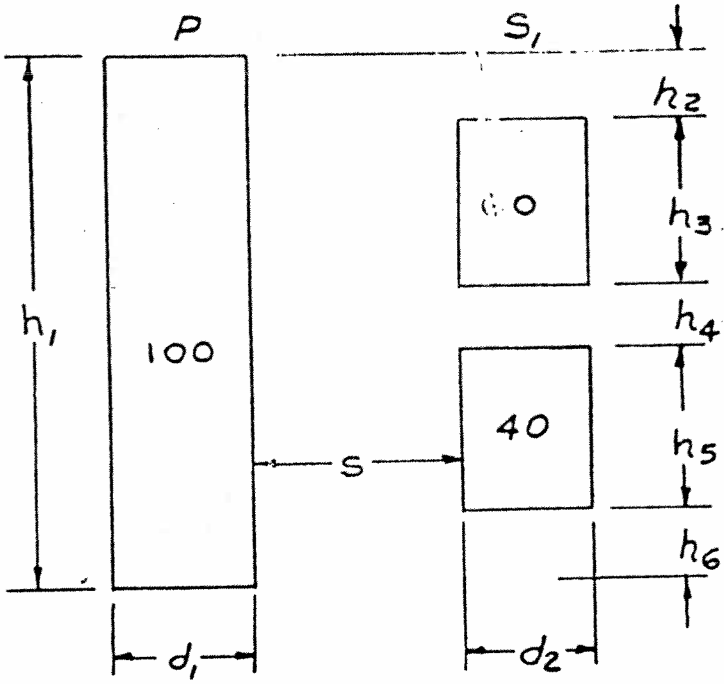


FIGURE 6.3-5

1. Assume the transformer is as shown in figure 6.3-6 and calculate the leakage inductance (L_L) using equation (6.3-7) or (6.3-8). Note that the assumed winding (S_2) has the same ampere turns as the actual winding (S_1) but uniformly distributed over the traverse (h_1) of the coil. The radial build of (S_1) and (S_2) is the same.

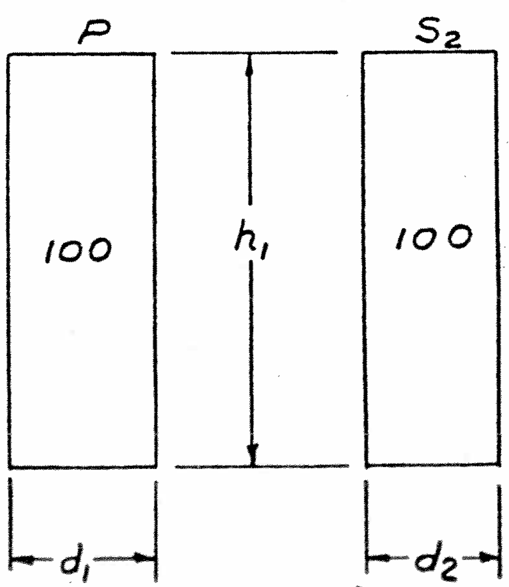


FIGURE 6.3-6

2. Draw the actual coil (S_1) as shown in figure (6.3-7) showing the ampere turn distribution over the traverse h_1. Short ends or tapped out sections will have (O) ampere turns as shown.

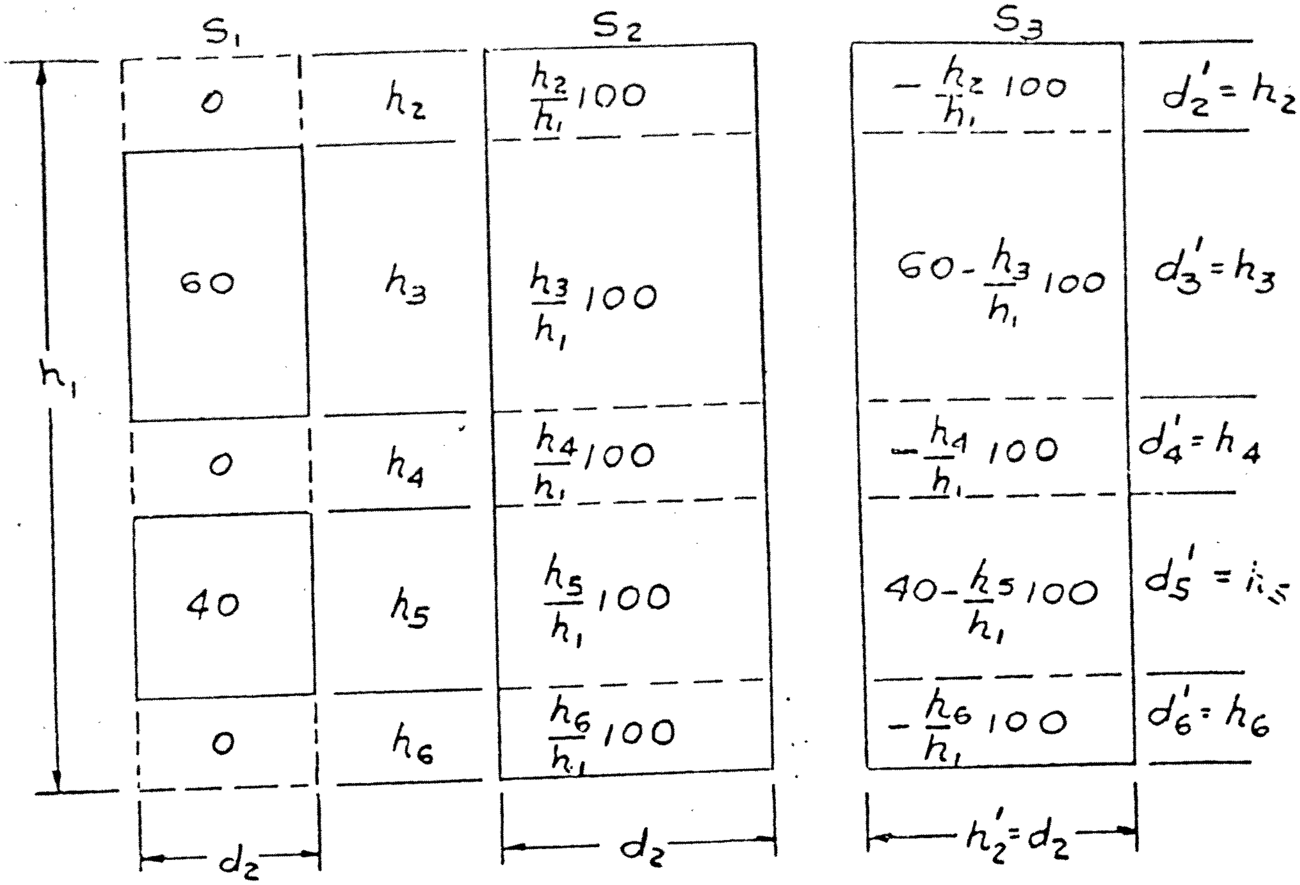


FIGURE 6.3-7

3. Draw (S_2) as shown in figure (6.3-7). Since (S_2) has its ampere turns uniformly distributed over the traverse (h_1) each section can be assigned ampere turns by the ratio of section traverse to the total traverse.

4. Draw a third winding (S_3), divided the same as (S_1) and (S_2) as shown in figure (6.3-7). The ampere turns of each section of (S_3) is the ampere turns of (S_1) minus (S_2). Note positive and negative ampere turns.

5. Calculate the leakage inductance (L_2) of (S_3), considering the negative sections as one winding and the positive sections as the other winding. Since there is no gap between windings, equations (6.3-5) can be used for this caclulation. Note that the coil build (d) is actually the traverse of each section and the traverse (h) is actually the build.

6. The total leakage inductance is the sum of (L_1) and (L_2).

$$L_2 = K \frac{\frac{.3^2}{3}2 + \frac{.3^2}{3}1}{.1} = K\frac{.03(3)}{.1} = K\frac{.09}{.1} = K(.9)$$

$$L_1 = K\left(.1 + \frac{.2}{3}\right) = K\,0.167 \qquad 5.38 \text{ times [illegible]}$$

6.3.1.5

The leakage inductance of a transformer having three or more windings can be determined from equations (6.3-7) or (6.3-8) by neglecting all but two windings. The process is repeated until the inductance between all possible paired combinations has been determined.

6.3.2 Distributed Capacitance

The total distributed capacitance of a transformer includes the capacitance between windings, between layers of a winding, from winding to the core, from winding to the tank, and the capacitance of all bushings. The designer must calculate or estimate each of the above capacitances, and refer them to a common voltage level to obtain the total distributed capacitance.

6.3.2.1

The following equations, taken from the National Bureau of Standards Circular C74, (8.1.5.4) will be found useful when calculating the distributed capacitance of a transformer.

The capacitance between two parallel plates is:

$$C_{DC} = 0.225 \frac{KA}{S} \quad \text{Picofarads} \qquad (6.3\text{-}9)$$

The capacitance between two concentric spheres is:

$$C_{DC} = 2.824\, K \left(\frac{R_1 R_2}{R_1 - R_2}\right) \quad \text{Picofarads} \qquad (6.3\text{-}10)$$

The capacitance between two coaxial cylinders is:

$$C_{DC} = \frac{0.6137\, hK}{\log \frac{R_1}{R_2}} \quad \text{Picofarads} \qquad (6.3\text{-}11)$$

Each of the above equations assumes that there is a uniform potential difference between the electrodes, and equations (6.3-9) and 6.3-11) neglect fringing at the edges of the electrodes.

6.3.2.2 Interwinding Capacitance

Since a uniform potential difference does not normally exist between two windings of a transformer, none of the above equations may be used directly.

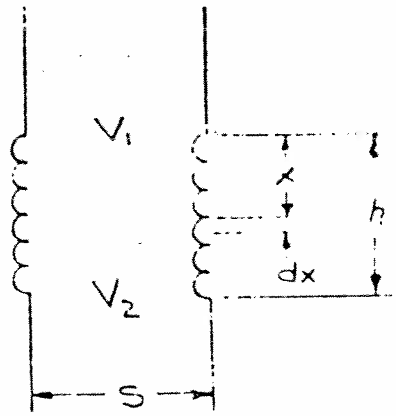


FIGURE 6.3-8

Figure 6.3-8 shows two adjacent windings or parts of windings with (v_1) volts between windings at one end and (v_2) at the other. If each winding has a uniform turn distribution, the voltage at X is:

$$v_x = v_1 + (v_2 - v_1)\frac{X}{h}$$

From equation (6.3-9) the capacitance of a small element (dx) wide and (ℓ_s) long is:

$$C_x = \left[0.225\,K\,\frac{\ell_s}{S}\right] dx \qquad \text{Picofarads}$$

The energy stored in this capacitance element is:

$$\mathcal{E}_x = \frac{1}{2}\,C_x\,v_x^2 = \left[\frac{0.225\,K\ell_s\,v_x^2}{2\,S}\right] dx \quad \text{joules}$$

$$\mathcal{E}_x = \frac{0.225\,K\ell_s}{2S}\left[v_1 + \frac{v_2X}{h} - \frac{v_1X}{h}\right]^2 dx \text{ joules (6.3-12)}$$

The total energy stored between the two windings can be found by integrating (6.3-12) from X = 0 to X = h.

$$\mathcal{E} = \int_0^h \frac{0.225\,K\ell_s}{2S}\left[v_1 + \frac{v_2X}{h} - \frac{v_1X}{h}\right]^2 dx \quad \text{joules}$$

$$= \frac{0.225\,K\ell_s h}{6S}\,(v_1^2 + v_1\,v_2 + v_2^2) \text{ joules}$$

$$= \frac{1}{2}\,C_d e^2 \text{ joules}$$

Where C_d is the distributed capacitance between the two windings referred to the winding of e volts.

Solving for C_d we get

$$C_d = \frac{.225\, K \ell_s h}{S} \left[\frac{v_1^2 + v_1 v_2 + v_2^2}{3 e^2} \right]$$

Since, for any given waveform

$$\frac{v}{e} = \frac{V}{E}$$

The above equation can be written

$$C_d = \frac{.225\, K \ell_s h}{S} \left[\frac{V_1^2 + V_1 V_2 + V_2^2}{3 E^2} \right] \text{ Picofarads} \qquad (6.3\text{-}13)$$

Comparing equation 6.3-13 with equation 6.3-9 we see that:

$$C_d = C_{DC} \left[\frac{V_1^2 + V_1 V_2 + V_2^2}{3 E^2} \right] \text{ Picofarads} \qquad (6.3\text{-}14)$$

For the special case where the voltage between windings is uniform, $V_1 = V_2$ and:

$$C_d = C_{DC} \frac{V_1^2}{E^2} \text{ Picofarads} \qquad (6.3\text{-}15)$$

Equation (6.3-15) can also be used to transfer a known capacitance value from one voltage V_1, to another voltage E.

6.3.2.3 Intrawinding Capacitance

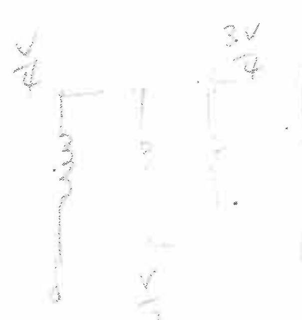
Figure 6.3-9 shows a conventional winding rated at E volts and consisting of m equal layers. The volts per layer is then:

$$E_L = E/m$$

Considering any two adjacent layers, the voltage between the two layers is zero at one end and two times the volts per layer at the other end.

Substituting in equation (6.3-14), the distributed capacitance between the two layers is:

$$C_d = C_{DC} \left[\frac{(0^2) + (0)(2E/m) + (2E/m)^2}{3 E^2} \right] = \frac{4\, C_{DC}}{3\, m^2}$$



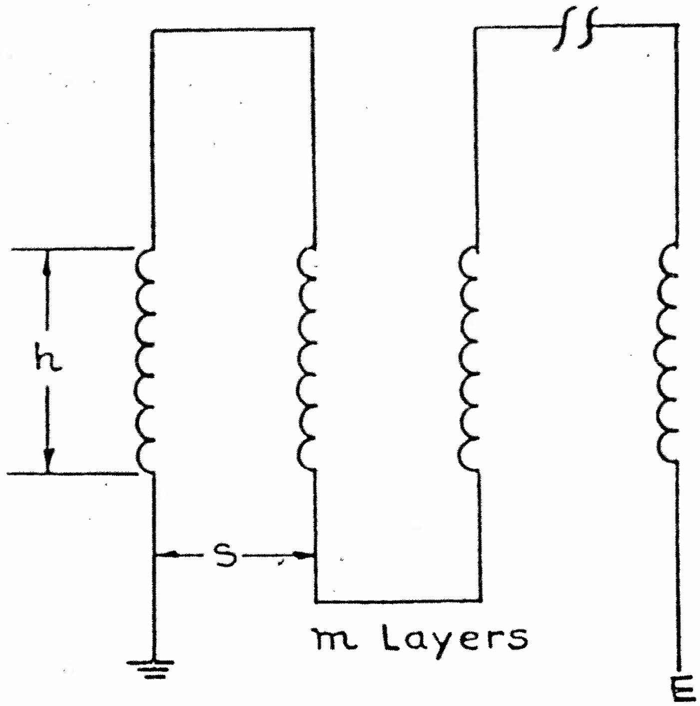


FIGURE 6.3-9

The total capacitance is (m-1) times the capacitance between two layers or:

$$C_d = 4C_{DC} \frac{(m-1)}{3m^2}$$

$.225\ \varepsilon_r \frac{A}{d} \Rightarrow .225 \frac{KA}{S}$

Substituting from equation 6.3-9 for C_{DC}

$$C_d = .30 \frac{K A}{S} \frac{(m-1)}{m^2} = \frac{0.30\ K \ell_s h}{S} \frac{(m-1)}{m^2} \qquad (6.3\text{-}16)$$

Equation (6.3-16) gives the intrawinding distributed capacitance of a winding, referred to that winding. Equation (6.3-15) can be used to refer this capacitance to any other winding.

Figure (6.3-10) shows another type winding, sometimes called a flyback winding, which is used to obtain low distributed capacitance.

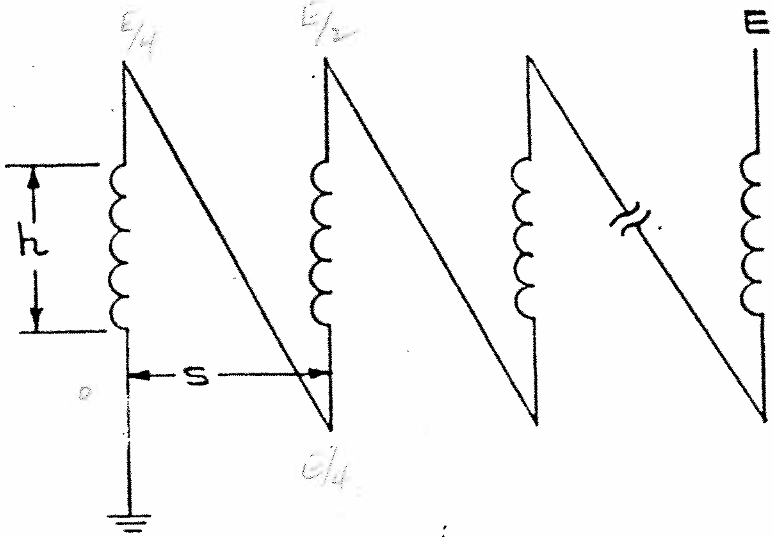


FIGURE 6.3-10

The volts per layer will be the same as for figure (6.3-9) but the voltage between layers is uniform and equal to the volts per layer. From equation (6.3-15) the distributed capacitance between any two adjacent layers is:

$$C_d = \frac{C_{DC}(E/m)^2}{E^2} = \frac{C_{DC}}{m^2}$$

The total capacitance is (m-1) times this value or:

$$C_d = C_{DC}\frac{(m-1)}{m^2}$$

Substituting from equation (6.3-9)

$$C_d = \frac{0.225\ K\ell_s h(m-1)}{S\ m^2} \quad \text{Picofarads} \qquad (6.3\text{-}17)$$

A comparison of equations (6.3-16) and (6.3-17) shows that the winding of figure 6.3-10 has 75% of the capacitance of the winding shown in figure (6.3-9).

6.3.2.4 Coil to Core Capacitance

The coil to core capacitance can be estimated using equations (6.3-9) and (6.3-14).

$$L_{L_p} = (1 - K^2) L_p$$

$$K = \sqrt{1 - \frac{L_{L_p}}{L_p}}$$